Problems - 4.1 - Theorem Proofs

Prove the following theorems. You may refer to other theorems as long as they have lower numbers.

(Thm 4.2) Suppose f is defined on open interval I, c is a real number and $g(x) = cf(x)$. If $f'(x)$ exists, then $g'(x)$ exists and $g'(x) = cf'(x)$.

(Thm 4.6) Suppose u and v are defined on open interval I, $v \neq 0$ on I and $f(x) = \frac{u(x)}{v(x)}$. If u' and v' exist, then $f'(x)$ exists and $f'(x) = \frac{u'(x)v(x) - u(x)v'(x)}{[v(x)]^2}$.

(Thm 4.10) Suppose f is continuous on open interval I, f takes on its maximum at x_0, and x_0 is an interior point of I (that is, x_0 is not one of the endpoints of I). If $f'(x_0)$ exists, then $f'(x_0) = 0$. (Hint: for $f'(x_0)$, do the left and right limits.)