Solutions - 4.2

(1) For the given function f, split the domain into intervals (as necessary) so that the f is either increasing or decreasing. For each interval, find the corresponding inverse function g.

$$f(x) = x^2 + 2x + 2$$

$f(x) = x^2 + 2x + 2 = (x + 1)^2 + 1$. We can see from the graph that f is decreasing when $x \leq -1$ and increasing when $x \geq -1$. A bit of algebra...

$$\begin{aligned} x &= (y + 1)^2 + 1 \\ x - 1 &= (y + 1)^2 \\ y &= \pm\sqrt{x - 1} - 1 \end{aligned}$$

If $x \leq -1$, then $g(x) = -\sqrt{x - 1} - 1$. If $x \geq -1$, then $g(x) = \sqrt{x - 1} - 1$.

(5) For the given function f, split the domain into intervals (as necessary) so that the f is either increasing or decreasing. For each interval, find the corresponding inverse function g.

$$f(x) = \frac{2x}{x + 2}$$

The function is not defined for $x = -2$, so we will split the interval there no matter what.

$$f'(x) = \frac{2(x + 2) - 1(2x)}{(x + 2)^2} = \frac{4}{(x + 2)^2}$$

Note that $f'(x) > 0$ on its domain, so f is increasing up to and after $x = -2$.

$$\begin{aligned} x &= \frac{2y}{y + 2} \\ xy + 2x &= 2y \\ y(x - 2) &= -2x \\ y &= \frac{-2x}{x - 2} \end{aligned}$$

On each interval, $g(x) = \frac{-2x}{x-2}$.