Solutions - 6.2

(9) (de Morgan, second part) If S is any space and F is a family of sets, show that

$$\left[\bigcap_{A\in F} A\right]^c = \bigcap_{A\in F} A^c$$

$$\begin{aligned} p \in \left[\bigcap_{A\in F} A\right]^c \quad &\Longleftrightarrow \quad p \text{ not in } \bigcap_{A\in F} A \\ &\Longleftrightarrow \quad p \text{ not in at least one } A \in F \\ &\Longleftrightarrow \quad p \in \text{ at least one } A^c \\ &\Longleftrightarrow \quad p \in \bigcap_{A\in F} A^c \end{aligned}$$

(10b) Let A, B be sets in space S. Define $A - B = \{p \in S : p \in A \text{ and } p \text{ not in } B\}$. Show that

$$A - (A - B) = A \cap B$$

$$\begin{aligned} p \in A - (A - B) \quad &\Longleftrightarrow \quad p \in A \text{ and } p \text{ not in } (A - B) \\ &\Longleftrightarrow \quad p \in A \text{ and either } (p \in B \text{ or } p \text{ not in } A) \\ &\Longleftrightarrow \quad p \in A \text{ and } p \in B \qquad \text{since } p \text{ can't be both in and not in } A \\ &\Longleftrightarrow \quad p \in A \cap B \end{aligned}$$

(11) In $\mathbb{R}^2$ with the Euclidean metric, find an infinite family of open sets $\{A_n\}$ such that $\bigcap A_n = \overline{B(0,1)}$, the closed ball of radius 1 centered at the origin. (Finding a family is sufficient, you don't need to prove that it works.)

The family $A_n = B(0, 1 + \frac{1}{n})$ for $n = 1, 2, 3, \ldots$ works.