Problems - 6.4

(4) Let l_2 be the space of sequences $x = \{x_1, x_2, \ldots\}$ such that $\sum_{i=1}^{\infty} x_i^2$ converges. Define $e_n = \{0, 0, \ldots, 0, 1, 0, \ldots\}$ where all terms are 0 except the nth term is 1. Define

$$d(x, y) = \left(\sum_{i=1}^{\infty} (x_i - y_i)^2 \right)^{\frac{1}{2}}.$$

Show that $\{e_n\}$ is bounded but not compact.