Solutions - 9.2

(4) Show whether the following series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n(n-\frac{1}{2})}$$

Since

$$\lim_{n\to\infty} \frac{\frac{1}{n(n-\frac{1}{2})}}{\frac{1}{n^2}} = 1$$

and since the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges as a p series, the given series is absolutely convergent.

(10) Show whether the following series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)\ln(n+1)}$$

Since $n+1$ and $\ln(n+1)$ are increasing to infinity, the absolute value of the terms are decreasing to 0. The series converges by the alternating series test.

However,

$$\int_1^{\infty} \frac{1}{(x+1)\ln(x+1)} dx = \ln(\ln(x+1))|_1^{\infty} = \infty$$

so with absolute values the series diverges. The given series is conditionally convergent.

(15) Show whether the following series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^n 3^{-n}$$

With absolute values we get a geometric series with $r = \frac{1}{3}$. Since $|r| < 1$, the series converges absolutely.