

The Problem Solving Competition - Problem #1 solution

Let $f(x) = x^2 - 30x + b$, where b is a constant. Suppose the maximum and minimum of $f(x)$ on the interval $[0, 20]$ have the same absolute value. Find b .

$f(x) = x^2 - 30x + b = (x - 15)^2 + b - 225$. The graph is a parabola, opening up, with vertex at $x = 15$. Since 15 is in the interval $[0, 20]$, the minimum $f(15) = b - 225$ occurs there. $f(0) = b$ and $f(20) = b - 200 < f(0)$, so $f(0) = b$ is the maximum. In order for $|b| = |b - 225|$ with $b > b - 225$, b must be positive and $b - 225$ negative. So $b = 225 - b$ and $b = \frac{225}{2}$.