

# The Problem Solving Competition - Problem #3 Solution

Find all real solutions to the following equation.

$$(\ln(2x + 3))^2 - 4(\ln(2x + 3))(\ln x) + 4(\ln x)^2 = 0$$

The left side of the equation factors.

$$\begin{aligned}(\ln(2x + 3))^2 - 4(\ln(2x + 3))(\ln x) + 4(\ln x)^2 &= 0 \\(\ln(2x + 3) - 2\ln x)^2 &= 0 \\ \ln(2x + 3) - 2\ln x &= 0 \\ \ln(2x + 3) &= 2\ln x \\ \ln(2x + 3) &= \ln x^2 \\ 2x + 3 &= x^2 \\ 0 &= x^2 - 2x - 3 \\ 0 &= (x - 3)(x + 1) \\ x = 3 \text{ OR } x = -1\end{aligned}$$

The original equation is undefined at  $x = -1$ .  $x = 3$  works though, so it is the only solution.