Research Statement
Paul Taylor

My interests lie in the broad field of Real Analysis. I am interested in all parts of analysis, although my research thus far has been in Fourier Analysis and Partial Differential Equations.

1 Fourier Analysis

The part of Fourier Analysis where I have done research is Oscillatory Integral Operators and Singular Integrals. These are integrals which have a $e^{i\rho(x)}$ term in the integrand and which would not converge without this term’s help. Since $e^{i\rho(x)}$ is changing sign all the time, it can help the rest of the integral converge if the positive and negative part cancel each other. For example, consider the following integrals:

\[
\int_0^\infty x^{-\frac{1}{2}} dx \\
\int_0^\infty e^{ix} x^{-\frac{1}{2}} dx
\]

The first integral diverges but the second integral converges thanks to the cancellation provided by the oscillating term $e^{ix}$.

The study of Oscillatory Integral Operators attempts to find efficient ways of using the cancellation to accurately estimate the size of an integral.

The specific integrals which I have studied are called Bochner-Riesz Means. These are integrals of the form

\[
T_{\lambda,t} f(x) = \frac{1}{(2\pi)^d} \int_{|\xi|<t} e^{ix \cdot \xi} \left( 1 - \frac{\rho(\xi)}{t} \right)^\lambda \hat{f}(\xi) d\xi
\]

where $\rho$ is an appropriate homogeneous distance function. These have been extensively studied in the case where $\rho(\xi) = |\xi|$, although that problem has not been completely solved. I study the case where

\[
\rho(\xi) = \max\{|\xi'|, |\xi_{d+1}|\} \quad \text{where} \quad \xi = \{\xi', \xi_{d+1}\} \in \mathbb{R}^d \times \mathbb{R}
\]

This presents interesting new results because the level sets $\rho(\xi) = 1$ are cylinders. Cylinders are not entirely smooth, unlike the spheres you get if $\rho(\xi) = |\xi|$. This difference in smoothness creates numerous complications.

My studies into this problem have thus far produced two papers, one accepted in the Transactions of the AMS [3] and one in collaboration with Sunggeum Hong and Chan Woo Yang which has been accepted by Mathematische Zeitschrift [2].
2 Partial Differential Equations

Since beginning my postdoctoral position with Eric Sawyer at McMaster University, I have begun to also work in Partial Differential Equations (PDE). The standard format of the problems on which we are working is to take an established result for PDEs where the coefficients are nice smooth functions, and extend it to similar PDEs where the coefficients are not smooth.

At this moment we are working to extend a renowned result of Hörmander on a PDE sometimes called the "sum of squares operator" [1]. There have been a number of prior extensions already, but they all made extremely rigid assumptions. We are hoping to open things up a bit.

References

