1. (7 points) Consider the vector-valued function

\[ r(t) = (\ln(t^2 + 1), t) \]

Set up a definite integral that describes the length of the path traced from \( t = 0 \) to \( t = 7 \). Do not evaluate the integral.

\[
\int_0^7 \sqrt{(x'(t))^2 + (y'(t))^2} \, dt
\]

\[
= \int_0^7 \sqrt{\left( \frac{2t}{t^2 + 1} \right)^2 + 1} \, dt
\]

2. (7 points) Find the point of intersection of the line with vector equation \( L(t) = (1, 2) + t(2, 1) \) and the line through the point \( (2, 4) \) that is perpendicular to \( L(t) \).

3. (7 points) Find the equation of a plane that goes through the point \( (1, -2, 3) \) and is perpendicular to the vector \( n = (-2, 1, 4) \). The equation should be written in the form \( ax + by + cz = d \).

\[ \mathbf{n} \cdot \langle x-1, y+2, z-3 \rangle = 0 \]

so \( -2(x-1) + 1(y+2) + 4(z-3) = 0 \)

\[ \Rightarrow -2x + y + 4z = 8 \]

The equation of the plane is: \[ ax + by + cz = d \]

\[ x = 1 + 2(t/4) = 13/5 \]

\[ y = 2 + 4t/4 = 14/5 \]

\[ z = t/4 \]

\( \therefore \) A point is \( (13/5, 14/5) \).
4. (7 points) Calculate $4k \times 3j$.

\[ = -12i \]

\[ \begin{vmatrix} j & k & l \\ 0 & 0 & 4 \\ 0 & 7 & 0 \end{vmatrix} \]

5. (7 points) Convert $(r, \theta, z) = (3, \frac{\pi}{6}, 4)$ from cylindrical to spherical coordinates.

\[ \Rightarrow \text{Rectangular: } \left( \frac{3\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 4 \right) \]

\[ \Rightarrow \text{Spherical: } \left( 5, \frac{\pi}{6}, \arccos \left( \frac{4}{5} \right) \right) \]

6. (7 points) Given the vector $\mathbf{v} = (-5, 12)$, find a unit vector in the same direction as $\mathbf{v}$.

\[ \| \mathbf{v} \| = 13 \quad \therefore \quad \frac{1}{\| \mathbf{v} \|} \mathbf{v} = \left\langle -\frac{5}{13}, \frac{12}{13} \right\rangle \]

7. (7 points) Set up a definite integral that describes how much area is enclosed within the cardioid $r = 1 - \cos \theta$ (with $0 \leq \theta < 2\pi$) shown below. Do not evaluate the integral.

\[
\frac{1}{2} \int_{0}^{2\pi} (1 - \cos \theta)^2 \, d\theta
\]
8. (7 points) Convert the following polar equation into an equation in rectangular coordinates.

\[ r = 3 \sin 2\theta \]

\[ r = 6 \sin \theta \cos \theta \]

\[ \Rightarrow r^2 = 6 \sin \theta \cos \theta \]

\[ \Rightarrow (x^2 + y^2)^{3/2} = 6y \cdot x \]

or \[ (x^2 + y^2)^2 = 36x^2y^2 \]

9. (7 points) Find the vector function \( \mathbf{r}(t) \) that satisfies \( \mathbf{r}'(t) = (\sin (t), \cos (2t), 200 - 32t) \) and \( \mathbf{r}(0) = (0, 10, 10) \).

\[ \mathbf{r}(t) = \left< -\cos(t) + c_1, \frac{1}{2} \sin(2t) + c_2, 210t - 16t^3 + c_3 \right> \]

Since \( \mathbf{r}(0) = \langle 0, 10, 10 \rangle \), this means \( c_1 = 1, c_2 = 10, c_3 = 10 \),

\[ \Rightarrow \mathbf{r}(t) = \left< 1 - \cos(t), \frac{1}{2} \sin(2t) + 10, 200t - 16t^3 + 10 \right> \]

10. (7 points) Complete the missing information in the table and sketch the curve \( r = 3 \sin 2\theta \) from \( \theta = 0 \) to \( \theta = \pi \).

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \pi/6 )</td>
<td>2.6</td>
</tr>
<tr>
<td>( \pi/4 )</td>
<td>7</td>
</tr>
<tr>
<td>( \pi/3 )</td>
<td>2.6</td>
</tr>
<tr>
<td>( \pi/2 )</td>
<td>0</td>
</tr>
<tr>
<td>( 2\pi/3 )</td>
<td>-2.6</td>
</tr>
<tr>
<td>( 3\pi/4 )</td>
<td>-3</td>
</tr>
<tr>
<td>( 5\pi/6 )</td>
<td>-2.6</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0</td>
</tr>
</tbody>
</table>
11. (10 points) Each of the following statements if FALSE. In each case, provide a specific example that illustrates that the statement is false.

(a) For all vectors u, v, and w, \( u \cdot (v \cdot w) = (u \cdot v) \cdot w. \)

\[ \text{For } \mathbf{u} = \langle 1, 2 \rangle, \mathbf{v} = \langle 1, 0 \rangle, \mathbf{w} = \langle 2, 1 \rangle \]

\[ u \cdot (v \cdot w) \text{ makeno sense b/c} \]

\[ \mathbf{u} = \langle 1, 2 \rangle \text{ and } v \cdot w = 2 \text{ and so } u \cdot (v \cdot w) \text{ doesn't exist.} \]

(b) For all vectors u and v, \( \|u + v\| = \|u\| + \|v\|. \)

\[ \mathbf{u} = \langle 3, 4 \rangle \text{ and } \mathbf{v} = \langle 1, 0 \rangle \]

\[ \|\mathbf{u} + \mathbf{v}\| = \|\langle 4, 4 \rangle\| = 4 \sqrt{2} \]

\[ \text{but } \|\mathbf{u}\| = \sqrt{5} \text{ and } \|\mathbf{v}\| = \sqrt{1}, \text{ so } 5 + 1 \neq 4 \sqrt{2} \]

12. (20 points) Choose TWO of the following problems and complete them below and/or on separate paper.

(a) The three points \( O(0, 0, 0), P(1, 2, 3) \) and \( Q(3, 1, 8) \) determine a unique plane that passes through the origin. Find the equation of this plane in the form \( ax + by + cz = d. \)

(b) It is possible for a pair of vectors u and v to satisfy \( u \cdot v = \|u \times v\|. \) What must be true about the angle between u and v for this to happen? Be very specific.

(c) The planes \( 3x + 2y + z = 7 \) and \( -3x + 2y - z = 9 \) intersect in a line. Find a vector equation for this line of intersection.

(d) The vertices \( A(1, 2, 3), B(3, 3, 5), \) and \( C(2, 0, 3) \) determine a triangle. Use the dot product to demonstrate that this is a right triangle, and then find the area of \( \triangle ABC. \)

(e) Prove that \( \|v + w\|^2 = \|v\|^2 + \|w\|^2 \) if and only if v is perpendicular to w.

(f) Prove that \( \frac{d}{dt} (r(t) \times r'(t)) = r(t) \times r''(t) \) for any twice-differentiable vector function \( r(t). \)