Section 14.8 Problems

**THEOREM 1 Lagrange Multipliers** Assume that $f(x, y)$ and $g(x, y)$ are differentiable functions. If $f(x, y)$ has a local minimum or maximum on the constraint curve $g(x, y) = 0$ at $P = (a, b)$, and if $\nabla g_P \neq 0$, then there is a scalar $\lambda$ such that

$$\nabla f_P = \lambda \nabla g_P$$

1. (Exercise 5) Find the max and min value of $f(x, y) = x^2 + y^2$ subject to the constraint $2x + 3y = 6$

2. (Exercise 7) Find the max and min value of $f(x, y) = xy$ subject to the constraint $4x^2 + 9y^2 = 32$

3. (Exercise 11) Find the max and min value of $f(x, y) = 3x + 2y + 4z$ subject to the constraint $x^2 + 2y^2 + 6z^2 = 1$
4. (Exercise 19) Find the area of the largest rectangle that can be inscribed in the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \).