Potential Integration Test Questions

1. Evaluate
\[ \int_1^3 \int_1^2 \frac{y}{x+y^2} \, dx \, dy \]

2. Sketch the region \( D = \{(x,y) : 0 \leq x \leq 2, 0 \leq y \leq 2x - x^2\} \), and then write \( \iint_D y\sqrt{x} \, dA \) as an iterated integral and evaluate it.

3. Find the center of mass of the sector of central angle \( \pi/3 \) (symmetric with respect to the \( y \)-axis) assuming that the mass density is \( \rho(x,y) = x^2 \). (See Figure 3 in the Chapter 15 Review Exercises.)

4. Prove the following formula for any continuous function \( f(x) \):
\[ \int_0^1 \int_0^y f(x) \, dx \, dy = \int_0^1 (1-x) f(x) \, dy \, dx \]
5. Evaluate

\[ \int_0^3 \int_1^4 \int_2^5 (x^3 + y^2 + z) \, dx \, dy \, dz \]

6. Use polar coordinates to set up \( \iint_D \sin (x^2 + y^2) \, dA \) where \( D = \{(x, y) : \pi/2 \leq x^2 + y^2 \leq \pi\} \) as an iterated integral. Do not evaluate it.

7. Set up an iterated triple integral in spherical coordinates for the integral of \( f(x, y, z) = x^2 + y^2 + z^2 \) over the region \( W = \{(x, y, z) : 1 \leq x^2 + y^2 + z^2 \leq 4\} \). Do not evaluate it.

8. Compute the line integral \( \int_C \mathbf{F} \cdot d\mathbf{s} \) for each of the following. Use the Fundamental Theorem of Line Integrals, if possible.

(a) \( \mathbf{F}(x, y) = \langle 2xy, x^2 + y^2 \rangle \) and \( C \) is the part of the unit circle \( x^2 + y^2 = 1 \) with \( x, y \geq 0 \), oriented counterclockwise.

(b) \( \mathbf{F}(x, y) = \left\langle \frac{2x}{x^2 + 4y^2}, \frac{8y}{x^2 + 4y^2} \right\rangle \) and \( C \) is the oriented path \( c(t) = \langle \cos t, \sin 2t \rangle \) for \( 0 \leq t \leq \pi \).
9. Let $C$ be the line segment from $(0, -1)$ to $(1, 1)$, and evaluate the line integral
\[ \int_C (x - y) \, ds \]

10. The plane $2x - y - z = 2$ is parameterized by $\Phi(u, v) = (2u + 1, u - v, 3u + v)$. Use a surface integral to find the surface area of $S = \Phi(D)$ where $D = \{(u, v) : u^2 + v^2 \leq 1\}$.

11. Use Green’s Theorem to evaluate
\[ \oint_{\partial D} (x + y) \, dx + (x^2 - y) \, dy \]
where $D$ is the region enclosed by $y = x^2$ and $y = \sqrt{x}$ for $0 \leq x \leq 1$.

12. Show how to use Green’s Theorem to find the area between $y = x^2$ and the $x$-axis for $0 \leq x \leq 2$. 