More Double Integrals

1. (15.2.10) Calculate the double integral of $f(x, y) = y^2$ over the region $\mathcal{R}$ defined as the rhombus with vertices $(0, 4), (0, -4), (2, 0)$, and $(-2, 0)$. (Draw a picture.)

2. (15.2.34) Sketch the domain of integration for $\int_0^1 \int_{e^x}^e f(x, y) \, dy \, dx$, and then express the equivalent iterated integral with the variables in the reversed order.

Triple Integrals

1. (15.3.5) Evaluate $\iiint_{\mathcal{B}} xe^{y-2z} \, dV$, where $\mathcal{B}$ is the box defined by $0 \leq x \leq 2$, $0 \leq y, z \leq 1$.

2. (15.3.15) Evaluate $\iiint_{\mathcal{W}} f(x, y, z) \, dV$, where $f(x, y, z) = z$ and $\mathcal{W}$ is the solid below the upper hemisphere with radius 3 but lying over the triangle in the $xy$-plane bounded by the lines $x = 1$, $y = 0$, and $x = y$. (Shown as Figure 12 in the textbook.)
3. (15.3.24) Find the volume of the solid in the first octant (i.e., \( x, y, z \geq 0 \)) bounded by \( x + y + z = 1 \) and \( x + y + 2z = 1 \). (You may use ContourPlot3D in Mathematica to see what this looks like.)

4. (Like 15.3.34) Find the total mass of the positive hemisphere with radius 3 if the mass density of the solid at point \((x, y, z)\) is given by the function \( \rho(x, y, z) = |x| \).

5. (15.3.35) Find the centroid of the tetrahedron with vertices at \((0, 0, 0)\), \((6, 0, 0)\), \((0, 4, 0)\), and \((0, 0, 8)\).