**Vector Operations Summary**

**DEFINITION Dot Product** The dot product $v \cdot w$ of two vectors
\[ v = \langle a_1, b_1, c_1 \rangle, \quad w = \langle a_2, b_2, c_2 \rangle \]
is the scalar defined by
\[ v \cdot w = a_1a_2 + b_1b_2 + c_1c_2 \]

**THEOREM 2 Dot Product and the Angle** Let $\theta$ be the angle between two nonzero vectors $v$ and $w$. Then
\[ v \cdot w = \|v\| \|w\| \cos \theta \quad \text{or} \quad \cos \theta = \frac{v \cdot w}{\|v\| \|w\|} \]

**THEOREM 3 Projection** Assume $v \neq 0$. The projection of $u$ along $v$ is the vector
\[ u_\parallel = (u \cdot e_v) e_v \quad \text{or} \quad u_\parallel = \left( \frac{u \cdot v}{v \cdot v} \right) v \]
The scalar $u \cdot e_v$ is called the component of $u$ along $v$.

**DEFINITION The Cross Product** The cross product of vectors $v = \langle a_1, b_1, c_1 \rangle$ and $w = \langle a_2, b_2, c_2 \rangle$ is the vector
\[ v \times w = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} i - \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} j + \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} k \]
where
\[ \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc \]

**Vector Operations Exercises**

1. Let $v = \langle 2, 5 \rangle$ and $w = \langle 3, -2 \rangle$.

   (a) Calculate $5w - 3v$

   (b) Find the unit vector in the direction of $v$.

   (c) Express $\langle 1, 1 \rangle$ as a linear combination of $v$ and $w$. 
2. Find the vector with length 3 that makes an angle of $\frac{3\pi}{4}$ with the positive x-axis.

3. Find a parameterization $\mathbf{r}(t)$ of a line passing through the points $(1, 4, 5)$ and $(-2, 3, -1)$.

4. Find the projection $\mathbf{w}$ of the vector $\mathbf{a} = \langle -1, 2, 3 \rangle$ onto the vector $\mathbf{b} = \langle 1, 1, 1 \rangle$. Demonstrate your answer is right by showing that $\mathbf{a} - \mathbf{w}$ is perpendicular to $\mathbf{b}$.

5. Let $\mathbf{v} = \langle 1, 3, -2 \rangle$ and $\mathbf{w} = \langle 2, -1, 4 \rangle$.
   
   (a) Compute $\mathbf{v} \cdot \mathbf{w}$

   (b) Use the previous answer to find the angle between the vectors $\mathbf{v}$ and $\mathbf{w}$.

   (c) Compute $\mathbf{v} \times \mathbf{w}$

   (d) Use the previous answer to find the area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$.

   (e) Use the dot product to demonstrate that $\mathbf{v} \times \mathbf{w}$ is perpendicular to both $\mathbf{v}$ and $\mathbf{w}$. 