Mobius Strips and Wild Topology

Shippensburg Area Math Circle
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The Mathemagician

Divide class in half: one group has me (the mathemagician). We are each going to cut a hole in this index card and without using any other materials we are going to create a hole in the card large enough for the basketball to easily pass through.

No tape. Only mathemagic!

The Topologist

Now, is this index card "the same" as it was before? How is it different? We will take some notes.

I do see it now has two kinds of edges- an inside one and an outside one. It has a "hole" on the middle. To a **topologist**, the index card is very different. In fact, a topologist would say two objects are the same if you can get from one to the other only by stretching, curving, but no cutting or gluing.

So, if I draw a circle on a piece of rubber paper then stretch it it looks like an oval. Well, the topologist would then say an oval and a circle are the same!!

But is a donut and a basketball the same? What's different?

Smart Board

Sorting at the smart board:
http://jwilson.coe.uga.edu/EMT668/EMAT6680.F99/Estes/unit/dayten/topology3.html

Is the original index card the same genus as the cut one?

Inside Outside

Let's each draw a shape. It has to start and stop at the same point but not cross itself. Go ahead, let's do it! Does this shape (on board) have an outside? an inside?

Here's something interesting: (line from "within" to outside hits the curve some number of times). Let's try with this fish puzzle and see if we can fine a way to always know outside from inside.

Back to warm-up problem: do we have any different thoughts now?

Mobius, oh my!

Whenever we play with topology one of my favorite things to think about is the Mobius strip. Let's each make one.

For each student: scissors, paper, tape, and pens (red and green) and pencils.

Activities

1. Create a Mobius strip. Trace a ladybug's path on it with your pencil. What makes this different from a simple loop? Make both if you need too.

2. What would happen if we cut a Mobius strip in half along that ladybug's path? Predict then try it.

3. What if two ladybugs were racing on the Mobius strip? Trace a red path and green path (equally spaced from edges). Now what would happen if we cut? Predict then try it!

4. Here's a sheet from a famous mathematician who loved puzzles and magic: Martin Gardner. In fact, we have a poster about him in the math department here. Let's make our predictions and see what happens!