

Platonic Solids<br>Shippensburg Area Math Circle<br>Lance \& Sarah Bryant, shipmathcircle345@gmail.com

Math Challenge: Make some Platonic solids and look for another formula.

What are Platonic solids? They are called that because the ancient Greek philosopher Plato wrote about them over 2,300 years ago. But that still doesn't tell you what they are. There is only one way to find out...

Here are some things to try:

1. Remember the pincushions we made by sticking blue struts along an equator of a zomeball? This time take the red struts and stick as many as you can into a zomeball. This shape is called a starburst.
2. Make sure your group makes a yellow starburst too.
3. For the red starburst stick a zomeball on the end of each strut. Do the same thing for the yellow starburst. Now start connecting the zomeballs together (except the one in the center of the starburst) with blue struts.
4. Alright, your starbursts should be fully surrounded by blue struts. Carefully remove the starburst (all the red or yellow struts and the center zomeball). Examine what is leftover after the starburst is removed. How would you describe it? I can tell you this much, it's a Platonic solid.
5. Your group now has two Platonic solids, one from red starburt and one from a yellow starburst. The one made out of pentagons is called a dodecahedron and the one made out of triangles is called an icosahedron. There are a total of five Platonic solids, but only three can be made with the blue struts. Can you make the third one? What would you call it?
6. Complete the table below.

| Platonic Solid | Dodecahedron | Icosahedron |  |
| :---: | :--- | :--- | :--- |
| Number of <br> Vertices |  |  |  |
| Number of <br> Edges |  |  |  |
| Number of <br> Faces |  |  |  |

This document is based on material from Zome Geometry by Hart and Picciotto.
7. For the dodecahedron, add the number of vertices and number of faces together. How does that compare to the number of edges?
8. Do \#7 again, but for the other two platonic solids that your group has built. If you let $v$ be the number of vertices, $f$ the number of faces, and $e$ the number of edges, can you write down a formula for what you have discovered?
9. If you have completed $\# 8$, then congratulations. You have just discovered a formula that was noticed by Leonhard Euler over 250 years ago. Of course, he didn't have the Zome system to play with. Go back to the table you made for prisms. Does your new "Euler's formula" work for prisms too?

