

# Polygon and Prism Formulas 

Shippensburg Area Math Circle
Lance \& Sarah Bryant, shipmathcircle345@gmail.com

Math Challenge: Figure out formulas for the construction of polygons and prisms.

If you look at a triangle made with the Zome system from far away, it would look like three points and three lines. In fact, it is a good idea to pretend that a zomeball is a point and a strut is a line when we are building, just like they would be if we were drawing shapes on paper. The points and lines that make up polygons and other figures have special names.

- A point of a polygon or other figure is called a vertex. The plural of vertex is vertices.
- A line of a polygon or other figure is called an edge.

Let's see if we can figure out formulas for the number of vertices and edges that polygons have, and then we will consider prisms.

Here are some things to try:

1. Use the polygons you have built to fill in the table below.

|  | Triangle | Square | Pentagon | Hexagon | Decagon |
| :---: | :--- | :--- | :--- | :--- | :--- |
| Number of <br> Vertices |  |  |  |  |  |
| Number of <br> Edges |  |  |  |  |  |

2. A chiliagon has 1,000 sides. How many vertices does it have? How many edges?
3. If an $n$-gon has $n$ sides, then how many vertices does it have? How many edges?
4. Alright, the answers you got in $\# 3$ are called formulas. You can replace $n$ with a number, for example 5, and then see how many edges a 5 -gon (or pentagon) has. Try it! Does it match the answer in the table from $\# 1$ ?

This document is based on material from Zome Geometry by Hart and Picciotto.
5. If we connect two polygons of the same kind as in the pictures below, it is called a prism. If we connect two triangles, it's a triangular prism. If we connect two squares, it's a square prism. If we connect two pentagons, it's a pentagonal prism. Well, you get the idea. With your group, build five different prisms, one for each kind of polygon that you have already built.

6. If we think of a prism as a box, then it has a bunch of sides all the way around it, and we will count the top and bottom as sides too. A side of a prism or other three-dimensional figure has a special name.

- A side of a prism or other three-dimensional figure is called a face.

Use the prisms your group has built to fill in the table below.

|  | Triangular <br> Prism | Square <br> Prism | Pentagonal <br> Prism | Hexagonal <br> Prism | Decagonal <br> Prism |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> Vertices |  |  |  |  |  |
| Number of <br> Edges |  |  |  |  |  |
| Number of <br> Faces |  |  |  |  |  |

7. How many vertices does a chiliagonal prism have? How many edges? How many faces?
8. How many vertices does an $n$-gonal prism have? How many edges? How many faces?
9. The answers you got in $\# 7$ are formulas. Replace $n$ with 4 and see how many faces a 4 -gonal prism (or square prism) has. Does it match the answer in the table from $\# 6$ ?
